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A primary concern of this study has been waveform design for active inverse problems of acoustic and electromagnetic variety. We developed several useful results in the specific areas of electromagnetic and acoustic bullets, signal design for doppler ultrasound velocimetry and magnetic resonance Imaging, and limited data tomography problems arising in medical imaging and in radar. Another area of proposed research involved the construction of maximum likelihood receivers for various novel signal sources, such as those arising in particular wideband acoustic data, and image data from non-eulidean sources. We have developed efficient algorithms in both of these regimes. In particular, we have studied receivers for application to wideband acoustic signal processing in acoustic velocimetry for the dense target environments occurring in Doppler ultrasound problems, and computationally efficient matched filter processor for the sphere. This has direct application to directional data of various forms, with applications from remote sensing problems to quality assurance for CAD/CAM.

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FINAL REPORT ON DARPA GRANT AFOSR-90-0292: RADAR, IMAGING, AND RELATED PROBLEMS

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Contents

1 Introduction

This note summarizes DARPA sponsored multi-disciplinary research at Dartmouth College under DARPA contract #AFOSR - 90-0292, during the period 1990-1993.

A primary concern of this study has been waveform design for active inverse problems of acoustic and electromagnetic variety. We developed several useful results in the specific areas of electromagnetic and acoustic bullets, signal design for doppler ultrasound velocimetry and magnetic resonance Imaging, and limited data tomography problems arising in medical imaging and in radar. These advances in waveform design for active remote sensing will be described in subsequent sections; we feel that they succesfully address one of the key points of our original proposal.

Another area of proposed research involved the construction of maximum likelihood receivers for various novel signal sources, such as those arising in particular wideband acoustic data, and image data from non-eulidean sources. We have developed efficient algorithms in both of these regimes. In particular, we have studied receivers for application to wideband acoustic signal processing in acoustic velocimetry for the dense target environments occurring in Doppler ultrasound problems , and computationally efficient matched filter processors for the sphere. This has direct application to directional data of various forms, with applications from remote sensing problems to quality assurance for CAD/CAM. These are also described more fully below.

In addition to these areas which were targeted in the original proposal, we have also studied post-processing enhancement schemes for noisy low contrast images. We have had very good results for medical images; in addition we have had expressions of interest by various groups concerning possible applications in problems of automatic target recognition.

In the following sections we survey our results in the research areas specified by the original proposal and also describe recent developments going beyond the scope of that proposal. The major results obtained during the course of the study fall into the three areas of theory, application, and computation. The discussion is organized correspondingly.

2 Theoretical Results

2.1 Wavelets and Coherent States

Theoretical work on discrete time Gabor frames may be found in [23]. A feature of this work is an algorithm for finding various good choices of the analyzing Gabor signals associated with a given family of synthesis Gabor waveforms.

A better understanding of the structure of wavelets can be obtained from new representations of the group SL(2,R). We obtained new representations of this group in terms of Jacobi modular forms [1], and in terms of Fourier Integral Operators [35].

Some theoretical work on coherent state methods can be found in [13]. This paper deals with a problem arising in quantum measurement theory: namely the informational completeness of certain families of operators in quantum systems admitting certain groups of symmetries. From the orthogonality theorem for square integrable representations on homogeneous spaces of a locally compact group, a series of lemmas are derived showing the informational completeness of natural covariant localization operators associated with the representations, as well as for the generalized Wigner distributions (matrix elements of the group). Some of these results give explicit reconstruction formulae for the quantum state from its expectation values against these families of operators. The results are applied to special (phase space) representations of the Heisenberg, Affine, and Poincaré groups.

2.2 Fast Fourier Transform Theory

We obtained algorithms for implementing Fast Fourier Transform algorithms for several interesting two- and three-dimensional geometries, including those invariant under the rotation group [7,14], and those corresponding to distance transitive graphs [8]. We also helped advise Harvard student D. Maslen in his extension of these results to general compact groups [29]. In the applications section, we will indicate various applications of these algorithms in statistical data analysis and quality control in computer aided manufacturing.

Another algorithm gives stable evaluation of the Fourier transform at non-uniformly spaced frequencies [31]. This algorithms is of special interest in fast magnetic resonance imaging applications.

2.3 Tomography

We obtained algorithms for producing tomographic images from incomplete data. In particular, one of these algorithms produces clear tomographic images from projection data restricted to certain angles of view in two dimensions (limited-angle data), a case of considerable interest in medical tomography, non-destructive testing [32,33,34], and radar [15].

Another algorithm addresses the local tomography problem [3], with interesting applications to reconstruction of moving objects from their projections [16].

2.4 Bullets

We obtained effective estimates for the decay rates of the relative error in the formation of acoustic and electromagnetic "bullets" [36]. These are solutions of the wave equation whose asymptotic form has compact support restricted to lie in a narrow cone. Such solutions are of substantial interest to the directed energy community. We have also shown that these bullet solutions can be used to construct a frame basis for the general compactly supported solutions of these wave equations. This frame basis has several useful properties. The motion of the basis members under the wave equation is easily described and explicitly given, and the expansion of the general solution in terms of the basis involves sampling on large spheres, for which Healy has already developed a fast algorithm.

3 Applications

3.1 Tomography in Medicine and Non-Destructive Testing

X-ray computed tomography (CT) is a very important medical imaging technique. We have considered regularization of two limited data inverse problems in CT.

Limited angle tomography is an imaging technique which must be used in many applied sciences, from geophysics to electron microscopy. It has been shown that the inversion of this problem is ill-conditioned. We have introduced an algorithm which decomposes the problem into tensor product structures to make it computationally feasible. Multiresolution techniques are used to stabilize the inversion. We prove that the appropriate Hilbert-Schmidt norms of the band-limited operators only grow linearly with respect to the discretization size. Most importantly, we recover images from limited angle data. [32,33,34]

Local Tomography refers to recovery of a sub-region of a density from a subset of the full line integral data normally taken in tomography. One major promise of local tomography is a reduction of radiation exposure in CT. We have introduced an algorithm which uses the properties of wavelets to essentially localize the dependence on x-ray data, [3]. We have demonstrated that the errors caused by the essential localization of the data are negligible when compared to the resolution of the imaging devices. Using these techniques, local reconstructions can be made with a reduction in data of up to 80%. We believe that this may prove to be a valuable tool in computerized tomography. Additionally, these techniques apply to Magnetic Resonance Imaging.

3.2 Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is an extremely versatile imaging modality, offering the ability to image motion and chemical effects in addition to excellent high contrast imaging of soft tissues. Nevertheless, it is often limited by restrictions in resolution and speed of image acquisition. Our principal efforts in this domain attempt to increase the flexibility and performance of existing hardware/procedures used for non-invasive diagnoses.

In recent papers [17,40] we discussed a first generation of applications of wavelet related techniques to signal acquisition in MRI. These provide specific benefits over traditional methods; among these are reduced imaging time and artifact reduction. However, this is accompanied by a lower signal to noise performance.

More recently we have noted that a priori information may often be used to enable one to create tailored bases for encoding signal features in such a way as to provide the resistance to artifacts only where it is needed, while encoding other portions of the image with more globally supported waveforms to enhance the signal to noise performance of the image acquisition in these regions [18,41].

Experimental studies of these ideas have been conducted, using the scanners at Dartmouth Hitchcock Medical Center. In addition, Dr. Larry Panych of Brigham and Women's Hospital wrote his MIT dissertation in part on implementation issues of our technique.

We have preliminary indications of the use of these methods for particular clinical tasks, including phosphorus spectroscopy, angiography, and for imaging moving anatomy, a notoriously difficult problem in MRI.

The structure theorems introduced in the study of local tomography are also of use in MRI imaging [16]. There are many instances when data cannot be gathered quickly enough to adequately form images of moving organs or to map the flow of blood. We use the structure of the Radon Transform to allow for optimal updating of projection type MR images as data is gathered. The structure theorems also allow for noise reduction, adaptive sampling, and image segmentation.

Beyond this we have been working to further speed the effective rate of acquisition of an MRI scanner by constructing optimal statistical estimators for the value of an image at a given, fixed time. Such "snap-shot" imaging techniques have performed well on simulated data. We are currently working on applying these techniques on a clinical MRI scanner. We anticipate that they will be more robust to motion artifacts, and may permit one to image certain moving structures more accurately than is possible with current techniques on standard clinical scanners.

3.3 Radar and Sonar

A fundamental goal of our research as laid out in our original proposal was to investigate the application of ideas from tomography in the deconvolution of radar ambiguity in dense target environment and other high resolution radar applications including SAR and ISAR. Our work on multiresolution approaches to limited data tomography has a direct and substantial impact on these problems, as we now indicate.

The basic notions in this area have been laid out in the work of Bernfeld, Grunbaum, Naparst, Speiser, and others [2,10,30,37,38]. Let us recall the narrowband situation for simplicity; when a dense target is probed with a waveform under the standard narrowband assumptions, the maximum likelihood estimate of the reflectivity is given by the matched filter expression, and represents a projection of the reflectivity into a space determined by the ambiguity of the waveform used. The researches mentioned above describe various means for using a diversity of radar waveforms to produce many different projections of the target reflectivity, in the attempt to obtain a more reliable reconstruction. A particularly simple approach uses signals with ambiguity approximating the knife edge, and amounts to recording strip integrals of the target reflectivity, as in tomography.

A particular problem in this approach is a standard limited data issue. Specifically, in several situations it may be difficult or imposible to get projection data in various angles of the range-doppler plane. For example, this may occur due to limitations on the obtainable chirp rates. This problem, identified specifically by Bernfeld and Grunbaum, is identical with the limited angle problem in tomography, which has been a very recondite problem indeed.

This problem has been attacked with new methods by our group with very good results. [34]. We are preparing a description of the method and its application to radar tomography. [15]

We have also been working with the problem of local tomography with application to imaging moving targets. To date this has been applied mostly in MRI, [16] but should find application in high resolution radar and sonar when the target motion cause significant evolution during imaging.

In addition, use of "bullet" pulses in radar problems would enhance considerably the effectiveness of the radar measurements. Efforts to describe and design an antenna to produce such pulses (the inverse source problem) are now underway.

3.4 Geometrical Quality Control in CAD/CAM

There are some interesting applications of our algorithms for efficient Fourier transforms on the sphere in the arena of statistical deconvolution problems arising with directional data. In recent work [11,12], these Fourier methods are used to compute a nonparametric estimate of some density of interest which has been obscured in the measured data by convolution with another density.

For example, in the geometric quality assurance application this confounding information is comprised of random orientation "noise" in an automated measurement process, and which may be described as coming from a density on $SO(3)$. Briefly, one wishes to determine from the automated measurement process whether or not a product is within spatial geometric tolerance to some ideal part designed through computer aided design (CAD). Earlier developments by Kim and others provide some statistical tests for geometric tolerancing. These depend on a Fisher-von Mises distributional assumption and that the CAD points are fixed and known. Our techniques provide some data analytic tools to check the Fisher-von Mises assumption and address the issue of variable CAD points.

3.5 Image Processing

These results concern processingand enhancement of images after acquisition. To date, our main applications have come with medical imaging. In this case, the particular type of processing applied to an image is determined by its ultimate diagnostic use; in general we have considered problems related to deblurring noisy images, and noise reduction while preserving salient image features.

In the deblurring application [25], we considered discrete space image recovery problems based on blurred noisy observations associated with a class of discrete blurring filters. A standard approach to regularizing the recovery of the underlying image uses a constrained least-squares fitting of the data with a roughness penalty derived from a conjugate highpass filter. In our work, the conjugate filter is taken to be a highpass wavelet filter associated with the blurring operator considered as a corresponding scaling filter. The resulting constrained least-squares regularization solution may be obtained iteratively using the associated wavelet reproducing kernel. In this way an explicit choice of regularizing operator is determined by the blurring operator. An iterative solution algorithm has been given, and numerical experiments performed on one and two dimensional data. The preliminary experimental results are at least as good as other common regularization techniques.

Our earliest work in filtering [44,45] describes a simple but effective nonlinear method for discriminating signal features from noise by correlating maxima of the wavelet transform modulus across scales. It works well, and has been illustrated with filtered MR images.

We now have a more sophisticated effort under way with Jian Lu, a former student and now a postdoc. The basic idea in this work [26] is the concept that the multiscale edges of an image determine a large portion of its content. This idea, originally put forth by Marr, has recently been elevated to the stature of a mathematical result by the works of Mallat and Zhong, Berman, and Zou. In our technique, the edges are abstracted from an image at a range of scales by finding the locations and amplitudes of the maximum moduli of a certain wavelet transform. These features are assembled into a hierarchical data structure, called the wavelet maxima tree. The edges in this tree connect related features at consecutive scales.

For noisy images, many of the branches in the tree correspond to edges induced by the noise. The emphasis is now on discriminating branches due to noise from those arising from actual image features. Scores for rating a branch have been developed; these scores reflect perceptual criteria such as the stability or persistence of a feature across many scales, and the stability or persistence of edge features within an extended region of space. These scores are used to set thresholds for pruning the data structure. The image is reconstructed from the surviving multiscale edges by means of a standard algorithm.

Analysis and experiment have demonstrated the effectiveness of these techniques in removing noise while preserving edge information in noisy images with low signal to noise ratio and low contrast. An extension of this idea is useful for contrast enhancement in various settings [27].

3.6 Feature Extraction for Recognition Tasks in Acoustic and Medical Data

We are currently investigating an extension of the classification of multiscale edge features in the tree representation beyond a simple discrimination of noise from signal. Specifically, we have been looking at a data set comprised of various acoustic recordings of helicopters, and another data set of mammography images. Together with J. Solka and Dr. C. Priebe of the Naval Surface Warfare Center in Dhalgren, Virginia, we will use our representation for extracting features to be used in various statistical classification algorithms under development by these scientists.

3.7 Ultrasound Doppler Velocimetry

We have discovered a new procedure for determining the velocity of fluids by means of ultrasound doppler measurements, using wavelet techniques to construct wideband maximum-liklihood receivers. This procedure is of special interest for determining the velocity of blood flow in pulmonary and cardiac medical problems.

Doppler ultrasound velocimetry is a non-invasive technique for measuring velocity profiles in blood vessels by scattering ultrasound off of blood cells and measuring the doppler shift. This technique holds the promise of permitting fine grained blood velocity mapping. This might be used, for example, to infer the existence of stenoses upstream. The current state of the art is limited by high variance in the estimates provided by the pulsed doppler techniques currently in use. One source of difficulty is due to the random variation of the phase arrangement of scatterers in the active volume from pulse to pulse (transit time effects).

The ultimate goal here is the construction of a map of the blood velocity in a region of a blood vessel. This is accomplished by means of careful measurements of the echoes of ultrasound pulses after they are scattered off of the red blood cells in that region. At present, Doppler ultrasound represents a very important non-invasive diagnostic tool. but refinements of the type we are considering are critical to its further development. For example, recent proposals for its use in diagnosis of stenoses by mapping regions of turbulent flow are very difficult to implement at the moment.

With Dr. Steve Jones of the Medical School at Johns Hopkins University, and former graduate student Joe DeStefano, we have tested techniques in simulation and actual experiment which significantly increase the statistical efficiency of the velocity estimates by deconvolving the transit time blurring. This work is presented in [4,5]. Some of these techniques are related to the deconvolution of radar and sonar ambiguity in dense target environment recently proposed in the narrowband case by Whitehouse, Speiser, et al. [42,37]. from whom we learned it [43]. Other narrowband tomographic processing ideas may be found in the work of Bernfeld [2] and Feig and Grunbaum [10].

A further project concerns the question: to what extent are the problems with conventional doppler ultrasound caused by inappropriate models? By relaxing the standard wide sense stationary uncorrelated scatters model of the scattering environment and using wideband models of the Doppler effect, we have analyzed some of the problems associated with the standard range gated techniques. We are investigating the signal design problem within this expanded context for further reduction of velocity estimator variance. In particular, we must compare our wideband estimators in doppler ultrasound to currently implemented techniques. This requires simulation and experiment on actual flow rigs, to be carried out with the assistance of Dr. Jones.

3.8 Matched Filtering

In addition, we have studied the idea of matched filters for various groups of symmetries. The idea here is a generalization of matched filtering in radar and sonar, where one searches for a signal in noise by correlating with a template. The signal is an echo of a known emission which has scattered from a target and will have suffered a time lag and a dilation, corresponding to the target's distance and its radial velocity. One estimates the lag and dilation parameters by correlating lagged and dilated versions of the template with the return, looking for the best match. This amounts to computing matrix elements of the affine group, acting on $L^2(\mathbb{R})$ by shift and dilation. Of course, one may wish to do this for other symmetry groups of a signal or image. For instance, in image processing one may wish to find a translated and rotated pattern by means of correlations over the Euclidean group of the plane.

Based on our FFT for the sphere, we now have an efficient algorithm for this sort of thing for data on the sphere [14]. We are currently searching for the generalizations to various symmetry groups.

For future applications of our work to signal processing we are looking at the very intriguing work of Holmes, Trachtenberg, [20,21,39] and others, who propose that nonabelian fast algorithms may be employed as potential approximants of the Karhunen-Loeve transform associated with a particular class of stochastic signals.

4 Computational Results

4.1 Scheduling for interval graphs

One of the most significant computational results so far comes from the algorithmic solution to the scheduling problem for interval graphs inherent in the direct Haar wavelet encoding MRI techniques described above. This solution leads to a general computational combinatorics result discussed in detail in [6]. Briefly, we consider the problem of ordering the dyadic intervals,

$$[\frac{j}{2^k}, \frac{j+1}{2^k}], \qquad 0 \leq j \leq 2^k - 1, \quad 1 \leq k \leq m$$

such that the separation in the ordering of intersecting intervals is at least some predetermined value. We prove bounds and provide an efficient algorithm to produce optimal schedules.

The generalization of this problem arises when we consider wavelets other than the Haar wavelet. At a given scale, general wavelets overlap some of their translates, unlike the Haar wavelet. We are applying optimization techniques to schedule the excitation of such wavelets, with penalties on overlaps constructed in terms of the tip angle function for the wavelet excitation.

4.2 Massively parallel wideband acoustic simulation

Our preliminary experiences with full wideband simulations in the area of Doppler ultrasound suggest that these pose a very heavy computational burden, which ultimately requires a parallel implementation. Our group has obtained good initial results with the C language on a DECmpp, with 2048 processors and 64K bytes memory each. Each simulation required the creation of 512K complex numbers, which were samples of the Fourier transform of a particular function. These numbers were then multiplied by the Fourier transforms of 256 other functions, and an inverse

FFT applied to the results. The large size of the tables required the use of cut-and-stack data storage and maximum-throughput FFT routines to achieve peak efficiency. More sophisticated simulations will require further development of this algorithm.

4.3 Stability of Fast Transforms on the sphere and experimental code

We have constructed a large, versatile implemenation of our FFT for the sphere. This has been used to demonstrate the stability and speed of the algorithm. We are now in the process of making this available to other groups, for their specific applications. For example, efficient algorithms for spherical harmonic expansions may prove useful in spectral methods for nonlinear coupled PDE in spherical coordinates such as those considered by Professor J. Bloxham's group in geophysics at Harvard in conjunction with their numerical studies of dynamo models of geomagnetism.

The computation of nonlinear terms in the equations describing the geodynamo involve a convolution in spectral space. The easiest way to do this is to transform from spectral to real space, multiply, then transform back. It is here that efficient algorithms can make a huge difference. This would also be the case for Atmospheric and Oceanographic Global Circulation Models (GCM).

We have been working with Martin St. Pierre of Bloxham's group in order to provide them with our code for evaluation on these sorts of problems.

4.4 Parallel Wavelet Code

One of our students has implemented an effective parallel wavelet transform for the Connection Machine. The results are described in [24]

4.5 Wavelet Edge Feature Code

The image processing code based on multiscale edge representation has been implemented on various platforms and in various languages. This project was partly supported under the present DARPA grant, and partly under a contract from the Naval Surface Warfare Center. The code is currently in use by our group and by our contacts at NSWC.

4.6 DSP

During the three year period of this study, we have succeeded in constructing in C and X-windows a general purpose digital signal processing platform called DSP. It implements nearly all of the signal processing and data processing techniques developed here. This platform has proved invaluable in testing and assessing various schemes and proposals for the practical implementation of the general theory.

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